STARTING ERRORS IN NUMERICAL APPROXIMATIONS TO AN EQUILIBRIUM DUSTY GAS MODEL

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ABSTRACT

Discontinuous initial data can generate spurious waves in the numerical solution of some systems of hyperbolic partial differential equations. In this paper we study a particular model of a dusty gas that appears to be especially susceptible to this type of starting error. Using a combination of analysis and numerical computation, we identify the source of the error and propose remedies. The method of analysis should be applicable to other systems.

1. INTRODUCTION

The interaction between a blast wave and the ground can stir up a large cloud of dust which can affect visibility and the performance of machinery. An attempt to model this behaviour using a set of equations first proposed by Marble, 1970, showed spurious solutions when the initial conditions include a discontinuous interface between a region heavily loaded with dust and a region containing no dust. The purpose of this study is to determine the cause of this starting error and to determine modifications to the equations and/or the numerical methods that will eliminate these errors.

2. DUSTY GAS EQUATIONS

Marble, 1970, derives a set of equations which describe the mechanics of a heterogeneous medium consisting of a dilute suspension of solid particles in an ideal gas. He assumes that the volume fraction of particles is so small that the interaction between individual particles can be neglected. For this study, we further assume that there is no slip between the particles and gas and that the particles and gas are in thermal equilibrium. This permits a reduction in the number of dependent variables. The resulting model still exhibits the error behavior that we wish to study.

The full paper contains a derivation of the model equations. The final result looks like the standard Euler equations applied to averages of the gas and particles plus an equation for an additional variable and a ideal gas equation of state with a variable specific heat ratio γ which is a function of the ratio of the particle density to the gas density, κ. Possible choices for the additional variable include κ or the gas density, ρ_g. We consider both conservative and non-conservative formulations of the equation for κ.

The result is a hyperbolic system of conservation laws of the form:

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{u})}{\partial x} = 0.$$

If the system were linear, its solution could be represented by waves with strength proportional to the eigenvectors of the matrix $\mathbf{A} = \partial \mathbf{f} / \partial \mathbf{u}$ travelling with speed proportional to the corresponding eigenvalues. The magnitude of the wave is equal to the projection of the variation of the solution on the corresponding eigenvector.

The wave like solutions to nonlinear problems are qualitatively similar but are more complex because the eigenvalues, eigenvectors and projections all depend on the solution. Roe, 1981; has shown that isolated nonlinear waves can be approximated by linearizations of the matrix **A** and that these linearizations can be used to construct numerical solutions.

3. RESULTS

In the full paper we show how , in this case, the starting error can be related to discontinuous initial conditions, nonlinearity, choice of dependent variables and choice of numerical method. In particular, we show how combinations of these factors contribute to the creation of spurious solutions. For example, if the initial condition is a jump in the variable κ, this variable is chosen as a dependent variable and it is discretized in a different way from the remaining equations, the nonlinear combination of the result of the energy equation and the κ equation generates a pair of pressure pulses as shown

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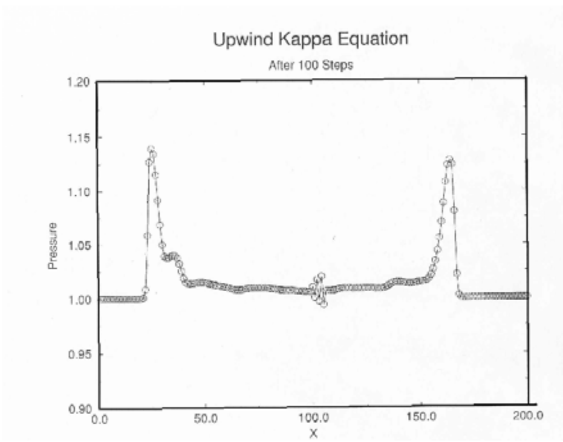
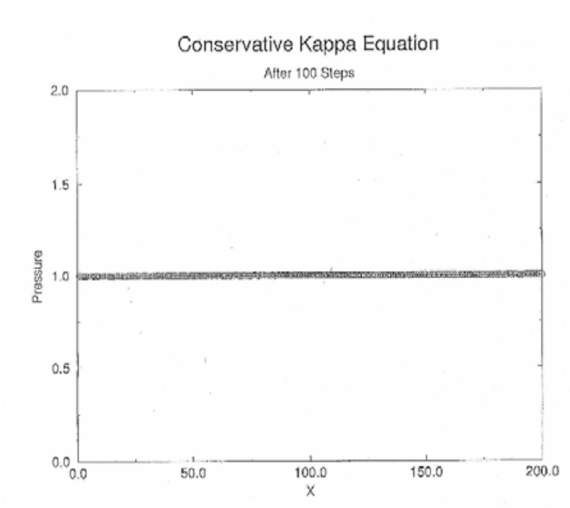
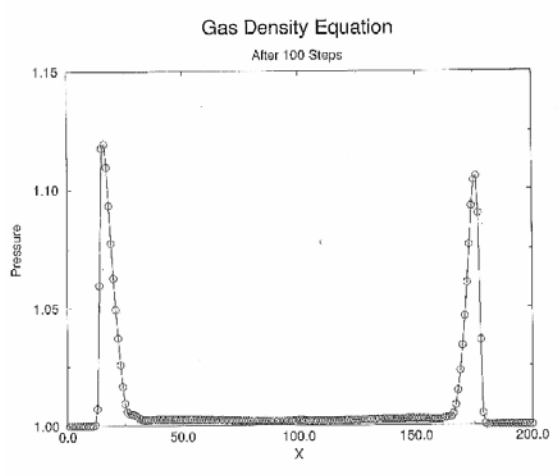
below. The pressure pulses have significant amplitude because the initial discontinuity is large.



On the other hand, if the κ equation is discretized in the same way as the remaining equations, the spurious pressure waves are not generated, as shown below.



However this is not the entire story. If ρ_g is chosen as the dependent variable in the last equation, spurious pressure waves seem to form regardless of how this equation is discretized. For the test case shown above, the artificial viscosity terms do not cancel when these variables are used to compute the pressure. This is shown in the following figure.



For the case of a κ discontinuity, spurious solutions can be characterized by pressure jumps. If the initial discontinuity is supposed to have no such jump, we can track the creation of such a jump to identify the source of error. This is done in the full paper for various choices of variables and numerical methods.

4. FUTURE WORK

For initial conditions involving other discontinuities, such as shocks, it is not obvious how to characterize spurious solutions. In the full paper, we suggest that the generation of spurious solutions can be related to the generation of spurious characteristic fields. In future work, this method will be extended to other cases where starting error is a problem, for example see Glaister, 1987; Karni, 1992; Noh, 1978; and Powell, Murman, Perez and Baron, 1987.

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